Complex Number 2

04 July 2023 18:34

Algebraic Representation of Complex Numbers

$+\sqrt{4} = +2$ $\qquad x^2 = 4$

$-\sqrt{4} = -2$ $\qquad x = \pm 2$

as x contains the sign

$z = (x, y)$

Real

$i = (0, 1)$

$z = \underline{3} = (3, 0)$

$(x, 0) + (y, 0) = (x+y, 0)$

$(0,1)(y,0) = (0 \cdot y - 1 \cdot 0, y + 0 \cdot 1)$
$= (0, y)$

$i^2 = (0,1)(0,1)$
$= (-1, 0) = -1$

$(0,1) \cdot (x,y) = (-y, x)$

$(x, y) = (x, 0) + (0, y) = x + (0,1) \cdot (y, 0) = x + iy$ (alg form)

Unique representation

$i^2 = -1$
$i = \sqrt{-1}$
$i^3 = i^2 i$
$= -1 i = -i$
$= -\sqrt{-1}$

$z_1 = x_1 + iy_1 \quad z_2 = x_2 + iy_2$

$$z_1 + z_2 = \underbrace{(x_1 + x_2)}_{\mathbb{R}} + \underbrace{i\underbrace{(y_1 + y_2)}_{\mathbb{R}}}_{\text{Imaginary}}$$

complex

$\mathbb{R}, \mathbb{I} \subset \mathbb{C}$

$(a+ib)(c+id)$
$= ac + ibid + ibc + iad$
$= ac - bd + i(bc + ad)$

$(a+ib)^2$
$= a^2 + (ib)^2 + 2aib$
$= a^2 + i^2b^2 + i2ab$
$= a^2 - b^2 + i2ab$

$\underset{Re(z)}{x} + i\underset{}{y} \rightarrow Im(z)$

$z \in \mathbb{C}$

$$\begin{cases} Re(z) = x = \dfrac{z + \bar{z}}{2} \\ Im(z) = y = \dfrac{z - \bar{z}}{2i} \end{cases}$$

Conjugate

$z = x + iy \qquad \bar{z} = x - iy$

$x = \dfrac{2x + iy - iy}{2} = \dfrac{2x}{2}$

$n \in \mathbb{N}$

$i^n \rightarrow n = 2m \rightarrow (i^2)^m = (-1)^m$ — m odd: -1; m even: 1

$i^n \rightarrow n = 2m+1$

i^{2m+1}
$= i^{2m} \cdot i$
$= (-1)^m \cdot i$

m odd: $-i$ $\qquad$ m even: i

$x = \dfrac{z + \bar{z}}{2} = \dfrac{x + iy + x - iy}{2}$

$y = \dfrac{z - \bar{z}}{2} = \dfrac{x + iy - (x - iy)}{2i}$

$z = \bar{z}$
$x + iy = x - iy$
$y = 0$

$\bar{\bar{z}} = x + iy = z$

$z \cdot \bar{z}$
$= (x+iy)(x-iy)$
$= x^2 + y^2$
$= |z|^2 \rightarrow$ modulus of z

$x_1 + iy_1$

$-i$

$= |z|^2 \rightarrow$ modulus of z

$z = (x, y)$ modulus of z is $\sqrt{x^2+y^2}$

$$\overline{z_1+z_2} = \overline{(x_1+iy_1+x_2+iy_2)}$$

$$= \overline{(x_1+x_2)+i(y_1+y_2)}$$

$$= (x_1+x_2) - i(y_1+y_2)$$

$$= x_1 - iy_1 + x_2 - iy_2 = \bar{z}_1 + \bar{z}_2$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$$\overline{\left(z \frac{1}{z}\right)} = \bar{1} = 1$$